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COUNTEREVASION STUDIES

Stephen S. Lane, et al

Texas Instruments, Incorporated

Prepared for:

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30 November 1974

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Depth Phase

Eigenspectrum

20 ABSTRACT (Continue on reverse side il necessary and identify by block number)

This report discusses progress to date on AFOSR Contract Number F44620-73-C-0055. The limit of the complex cepstrum's resolution when separating depth phases is found, and it is found that the eigenspectral technique is impractical in the general case.

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COUNTEREVASION STUDIES

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SECTION I

This report discusses the continuation of the work described in Semi-Annual Technical Report No. 2 - Part B (Lane and Sun, 1974). In that report the complex cepstrum and eigenspectrum techniques were applied to synthesized real interfering signals, and a possible multiple explosion was analyzed by the complex cepstrum technique. In this report the ability of the complex cepstrum technique to resolve depth phases from presumed underground nuclear explosions is investigated, and it is shown that the eigenspectrum technique is unworkable in the general case.

SECTION II CEPSTRUM ANALYSIS

A. INTRODUCTION

In Semi-Annual Technical Report No. 2 - Part B, a detailed description of the complex cepstrum was given, and the effect of exponentially weighting the input trace was discussed from a theoretical point of view. The limit of the cepstrum technique as applied to synthetic mixtures of underground nuclear explosions was found to be about 0.5 seconds, and an example of a presumed multiple underground nuclear explosion was analyzed.

The purpose of the present report is to develop a more effective form of comb filter for the cepstrum, and to investigate the limit to which the depth phase from underground nuclear explosions can be resolved. A matched waveform filter has been developed to aid in the solution of these problems.

Subsection B discusses the theoretical basis for the comb and matched waveform filters. Subsection C is concerned with synthetic mixtures of direct and depth phases from presumed underground nuclear explosions.

B. COMB FILTERING OF CEPSTRA

It is necessary, for proper filtering of a cepstrum due to mixed signals, to place the filter at the correct point in time. It is also important to use the correct type of filter. For delay times of 0.6 seconds or greater a short pass filter, which zeros all points in the cepstrum corresponding to delay times greater than the filter point, has been found to be most useful.

In this report, a comb filter is developed which sets the cepstrum to the value it would have had in the absence of a multipath operator.

Let the digitized signal s(n) from a direct P wave be combined with a scaled and delayed replica of itself to simulate a signal from an underground nuclear explosion

$$x(n) = s(n) + a s(n - N)$$
 (II-1)

where a is a scale factor and N is a delay time in units of the sampling time.

The z transform of equation II-1 is

$$X(z) = S(z)(1 + az^{-N})$$
 (II-2)

where S(z) is the z transform of the signal. The logarithm of the z transform is

$$\hat{X}(z) = \ln S(z) + \ln(1 + az^{-N})$$
 (II-3)

and the complex cepstrum $\hat{x}(n)$ is the inverse Fourier transform of equation II-3. If

$$|a| < 1 \tag{II-4}$$

which was shown in Semi-Annual Technical Report No. 2 to be the condition that the multipath operator be minimum phase, then the right hand side of II-3 can be expanded in a Laurent series as

$$\hat{X}(z) = \ln S(z) + \sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{az^{-N}}{k}\right)^k$$
 (II-5)

The cepstrum of the observed signal is the inverse transform of equation II-5.

$$\hat{x}(n) = \hat{s}(n) + \sum_{k=1}^{\infty} (-1)^{k+1} \frac{a^k}{k} \delta(n - kN)$$
 (II-6)

and the value of the kth cepstral peak due to the multipath operator alone is

$$\hat{m}_{k} = (-1)^{k+1} \frac{a^{k}}{k}$$
 (II-7)

When exponential weighting α^n is employed (i.e., the nth time point is weighted by α^n) it is easy to show that the kth cepstral peak of the multipath operator, at time kN, becomes:

$$\widehat{m}_{k} = (-1)^{k+1} \frac{(a\alpha^{N})^{k}}{k}. \qquad (II-8)$$

Thus to find the cepstrum due to the signal alone we must set the observed cepstrum to its value minus the value of the multipath operator found in II-8.

For times larger than the reciprocal of the signal bandwidth the only contributions to the cepstrum are due to the multipath operator and to noise. Therefore the best filter is a combined comb and short pass filter, which adjusts the cepstrum by means of II-8 for short times, and sets it equal to zero for long times. For delay times larger than the signal bandwidth reciprocal all the cepstral peaks are separated from the peak due to the signal and may be removed with a short pass filter, as in Semi-Annual Technical Report No. 2.

The discussion of Semi-Annual Technical Report No. 2 concerning the mixture of non-identical signals is applicable here. If the mixed signals have the same approximate distributions of zeros in their z transform the comb filter will resolve them satisfactorily.

To resolve mixed events using this filter we must know not only the time delay between them but also their amplitude ratio. Such information almost solves the problem of resolving interfering events, except for the exhibition of the waveforms involved. Since displaying these waveforms is such a crucial part of the cepstrum technique it is legitimate to seek some independent method of finding the delay times and amplitudes of the interfering surface reflection.

Matched filtering has proved to be useful for this purpose. A reference waveform was constructed by selecting a presumed explosion waveform with no discernable depth phase. This waveform was added to a scaled replica of itself with a time delay as in equation II-1. The zero time delay cross-correlation function was formed between this reference waveform and the waveform from the event under study for various values of scale factor and time delay for the artificial depth phase. The parameters yielding the maximum cross-correlation were taken to be the amplitude ratio and time delay of the unknown depth phase. Sharp maxima were found when the time delay was varied, and broader peaks were found with variation of the scale factor.

C. EXPERIMENTAL RESULTS

Three presumed underground nuclear explosions from eastern Kazakh as recorded at NORSAR were selected for analysis, to find the minimum time at which a depth phase could be resolved. Each was filtered with the complex cepstrum, and successfully resolved into a direct arrival and a depth phase. The depth phase occurred about 0.6 seconds after the first motion in each case, and its amplitude varied from 0.9 to 1.1 times that of the direct arrival. Each depth phase was then added to the corresponding direct arrival with delay times from 0.1 to 0.6 seconds, keeping the amplitude ratio fixed. In this way signals with a realistic degree of dissimilarity between direct and depth phases were formed.

Each signal thus synthesized was subjected to analysis by the matched waveform technique. In every case the time delay was predicted correctly, and the amplitude ratio was predicted with reasonable accuracy. Using these time delays and amplitude ratios, the synthesized signals were analyzed with the complex cepstrum. Every mixed signal was resolvable into a direct arrival and depth phase down to a time delay of 0.1 seconds.

Similarity between the resolved and original waveforms was good. An example is shown in Figure II-1, where an event from Kazakh has been resolved into a direct arrival and depth phase, having an amplitude ratio of about 1.2. These phases were recombined at a time delay of 0.3 seconds, as shown in Figure II-2. The output of the matched waveform filter is shown in Figure II-3 for an amplitude ratio of 1.1. The maximum occurred at 0.3 seconds, and the cepstrum filter was applied there. The output waveforms shown in Figure II-4 are clearly similar to those used to synthesize the mixed waveform.

Next matched waveform filtering was applied to four small presumed explosions from Kazakh, recorded at NORSAR. Delay times between P and pP from 0.5 to 0.2 seconds were found, and the cepstra were filtered at those times. At the longer delay times satisfactory resolution of the signals was achieved, but filtering the signal predicted to be delayed by 0.2 seconds did not yield two similar signals.

The second highest correlation coefficient for this signal was found at a delay time of 0.1 seconds. A cepstrum filter applied there did produce two similar signals. The signal-to-noise ratio of this event was the lowest of those considered, and it is reasonable to suppose that noise influenced the estimation of the delay time by matched waveform analysis. This was not the case for the synthetic signals—since they were formed from events recorded at high signal-to-noise ratio.

From these results we conclude that there is no difficulty attributable to the cepstrum technique itself in extracting depth phases at delay times down to 0.1 seconds. Difficulties may arise with naturally recorded events at small delay times because these events also have low signal-to-noise ratio, which leads to difficulties with the cepstrum analysis at any delay time.



Time in Seconds

(a)

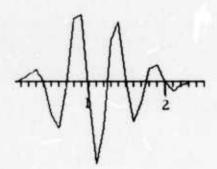


Time in Seconds

(b)

FIGURE II-1

DIRECT ARRIVAL (a) AND DEPTH PHASE (b) FROM PRESUMED NUCLEAR EXPLOSION



Time in Seconds

FIGURE II-2
RECOMBINED DIRECT ARRIVAL AND DEPTH PHASE

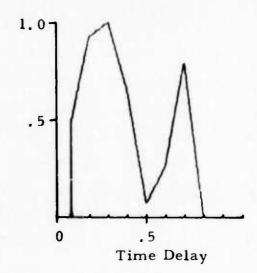
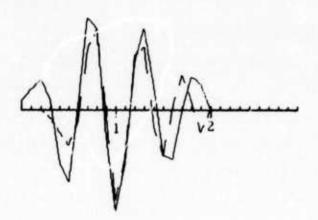


FIGURE II-3
MATCHED WAVEFORM FILTER OUTPUT



Time in Seconds

Direct Arrival

----- Depth Phase

FIGURE II-4
RESOLVED WAVEFORMS

For the sake of completeness the cepstrum technique was applied to a number of large events from eastern Kazakh, ranging in magnitude from 5.5 to 6.0. In each case the cepstrum extracted a clear depth phase from the recorded trace. Table II-1 shows the delay times found for these events and for the smaller ones analyzed earlier, together with their magnitudes as estimated from Lambert (1969). Also shown is the delay time calculated by assuming that the events are 'normally buried' at a depth in feet of $H = 400 \cdot W^{1/3}$, where W is the yield in kilotons of TNT. Twice this depth divided by an assumed P wave velocity of 3.2 km/sec is then the expected delay time.

The agreement between these independent means of estimating the delay time is good for the large magnitude events, especially in light of the fact that a good relation between magnitude and yield for these events is not known, and that the P wave velocity has been estimated rather than measured.

The delays calculated by cepstrum analysis and cube root scaling for the two smallest events of magnitude 4.5 and 4.9 disagree significantly. Two explanations for the observed discrepancy for the magnitude 4.5 event are possible. If this event was a cratering explosion the time delay would be less than that predicted on the basis of containment, as observed. It is more likely, however, that seismic noise interferes with the cepstral analysis to produce an incorrect estimate of the delay time for events of such small magnitude. This explanation is the only one possible for the discrepancy in the case of the magnitude 4.9 event, and is supported by the fact that matched waveform analysis for this event gave a delay time of 0.2 seconds, in good agreement with that from cube root scaling. In any case, these examples show that cepstral analysis is uncertain for real signals at delay times of 0.1 or 0.2 seconds.

TABLE II-1
ESTIMATED DELAY TIMES FOR VARIOUS EVENTS

Event Magnitude	Estimated Yield	Delay Time (seconds) From	
m _b	(kilotons of TNT)	Cepstrum	Yield
4.5	20	0.50	0.21
4.9	40	0.10	0.26
5.5	200	0.50	0.44
5.5	200	0.50	0.44
5.5	200	0.60	0.44
5.5	200	0.60	0.44
5.6	250	0.40	0.47
5.7	310	0.70	0.53
5.8	400	0.60	0.58
5.9	520	0.70	0.67
6.0	650	0.90	0.78

D. CONCLUSIONS

From this study we conclude that the combined comb and short pass filter used in the complex cepstrum technique is the best filter to use for time delays less than about 0.6 seconds. The complex cepstrum technique using this filter and matched waveform analysis may be used to resolve depth phases from presumed underground nuclear explosions down to delay times of 0.4 seconds. Estimates of shorter delay times may be made with increasing uncertainty in their accuracy.

SECTION III EIGENSPECTRAL ANALYSIS

A. INTRODUCTION

Semi-Annual Technical Report No. 1 (Lane, 1973) presented a detailed explanation of the eigenspectral technique for resolving interfering long-period signals. Semi-Annual Technical Report No. 2 - Part B, (Lane and Sun, 1974) showed that is was possible, in some cases, to resolve interfering signals separated in power by 20 dB through eigenspectral analysis. Further research has shown that such results are the exception rather than the rule. In this section an example is presented using nearly ideal data which illustrates the difficulties encountered and suggests their origin. These problems are inherent in the use of real data, and appear to make the eigenspectral technique unworkable in the general case.

B. EXPERIMENTAL RESULTS

As an example in which any problems encountered could be attributed to the technique rather than the data a large event from the Tonga Islands was mixed with a smaller scaled event from Crete. Both were recorded at ALPA at a rate of 2 seconds per data point. The Tonga Island signal followed an oceanic path, and was free of multipathing effects. It was large enough that seismic noise for the combination was well below the scaled Crete event for reasonable power separations. The great circle azimuth to the Tonga Island event was 202° and to the Crete event was 5° so that no problems with angular resolution should have been encountered. Cross spectral matrices were formed from the data vectors and averaged over 21 frequencies

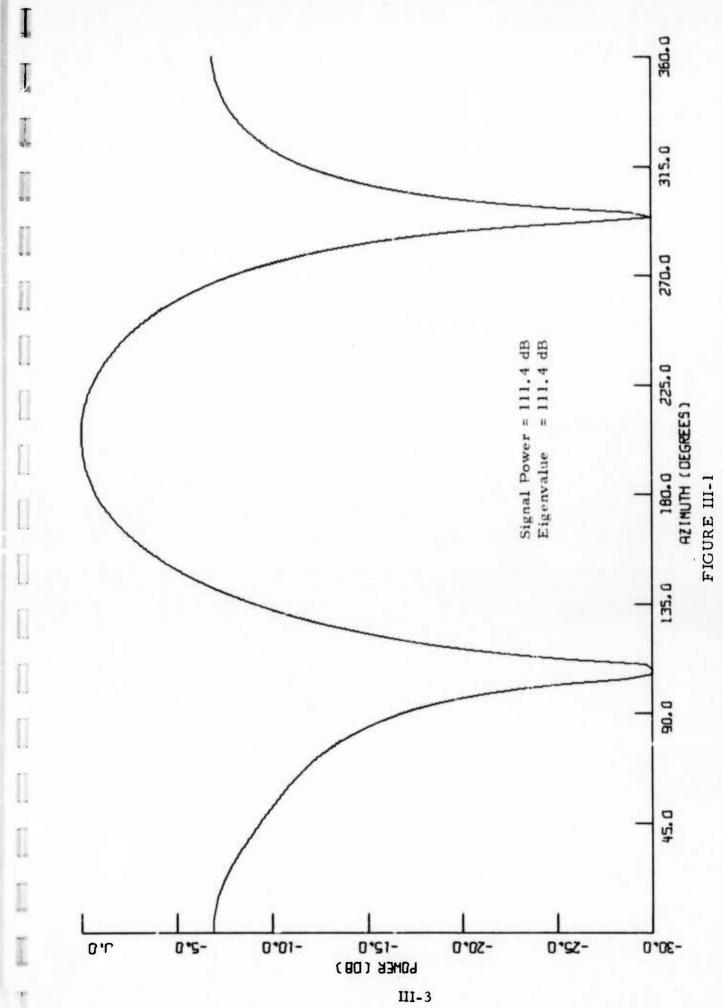
from . 0425 Hz to . 0471 Hz; power equalization between channels was excellent.

To find what results should be expected if the signals were resolved perfectly, cross power matrices for the Tonga Island and Crete events were first formed separately. Their first eigenspectra are shown in Figures III-1 and III-2, respectively. Each shows a distinct peak at almost the azimuth of the great circle path as it should. The second eigenspectrum of the Tonga Island event alone appears in Figure III-3, and shows a peak at 288°, about 90° from the peak of the first eigenspectrum at 204° and 24 dB below it.

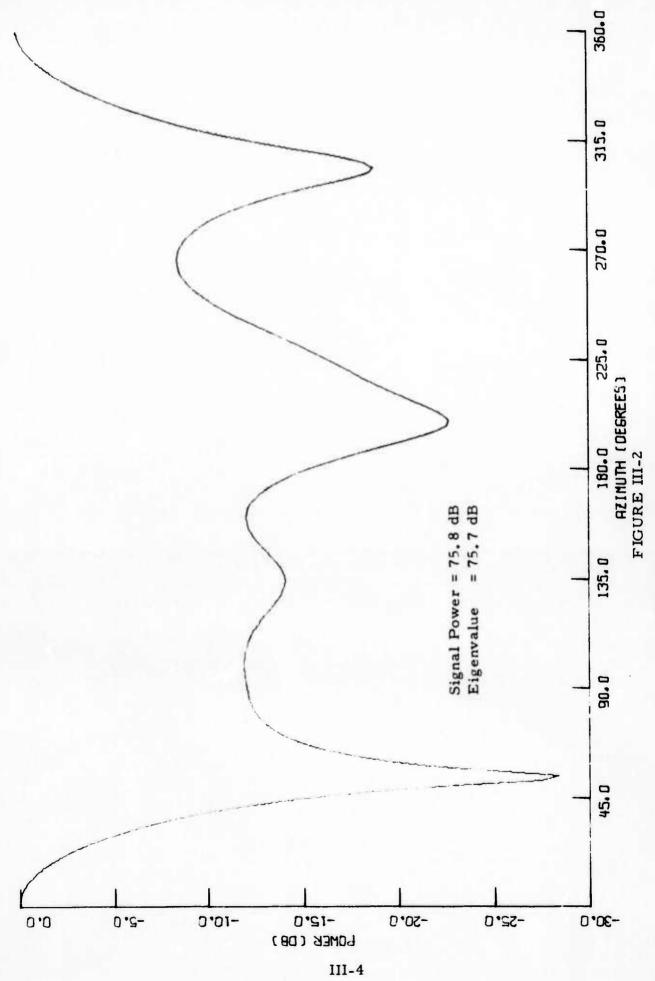
Next the events were mixed together by adding their data vectors, with a scale factor for the Crete event, and averaging their data matrices over frequency as before. The first eigenspectrum for the mixture of events is almost identical to that of Figure III-1 for the power separations considered and is not shown here. In Figures 11I-4 through III-6, the second eigenspectrum of the mixture of events is shown for various power separations. Where the Crete event is 12 dB below the Tonga Island event, in Figure III-4, the azimuth of the second eigenvector is that of the Crete event, but the second eigenvalue is 10 dB below the Crete event's power, certainly an unacceptable error. This error is caused by inclusion of some of the second signal's power in the first eigenvalue where it makes a negligible error. The effect of its loss on the smaller signal is substantial, however.

When the Crete signal is 18 dB below the larger event, in Figure III-5, the peak of the second eigenspectrum lies at 310° rather than near 0°. This error is due to the interference of the peak at 288° found in the second eigenspectrum of the Tonga Island event.

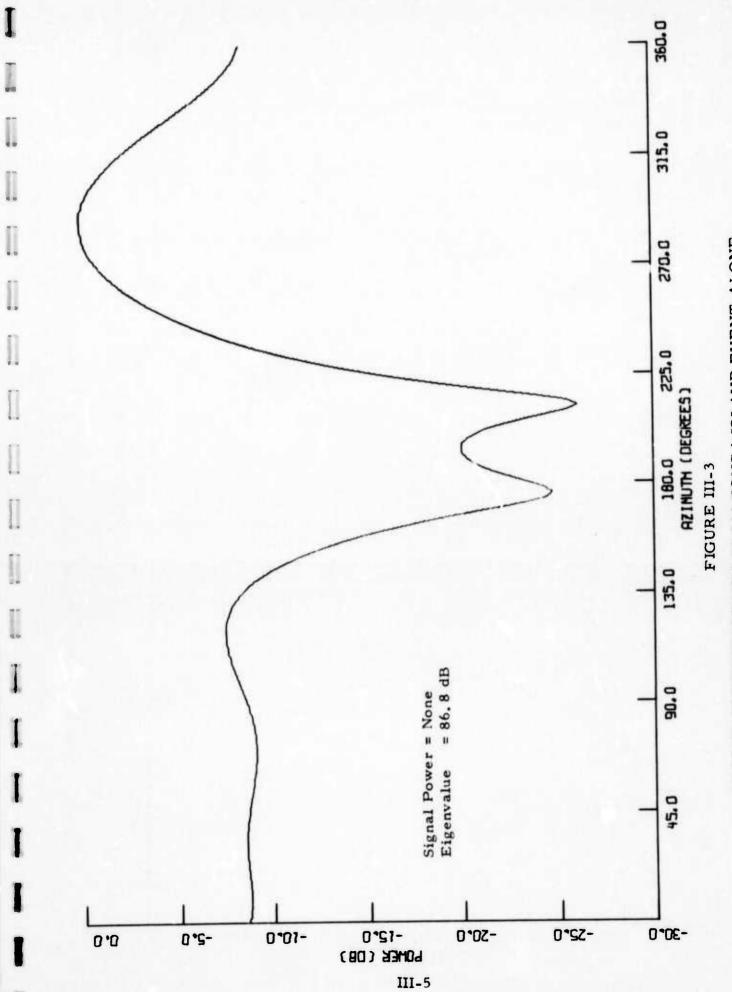
For still lower Crete powers, as in Figure III-6 where the power separation was 24 dB, the second eigenspectrum is almost the same



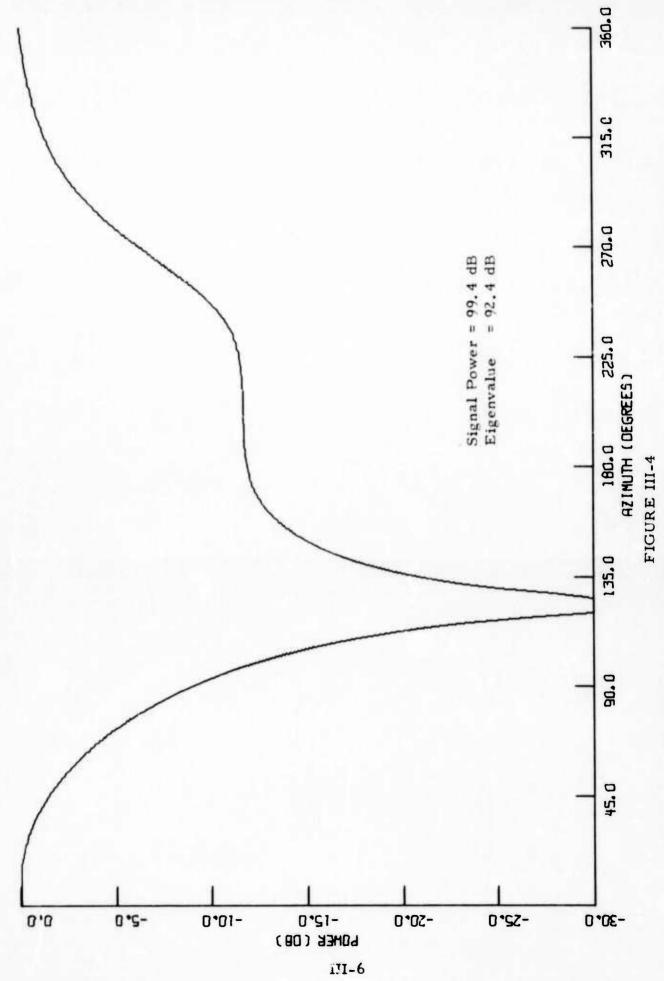
FIRST EIGENSPECTRUM FROM TONGA ISLAND EVENT ALONE



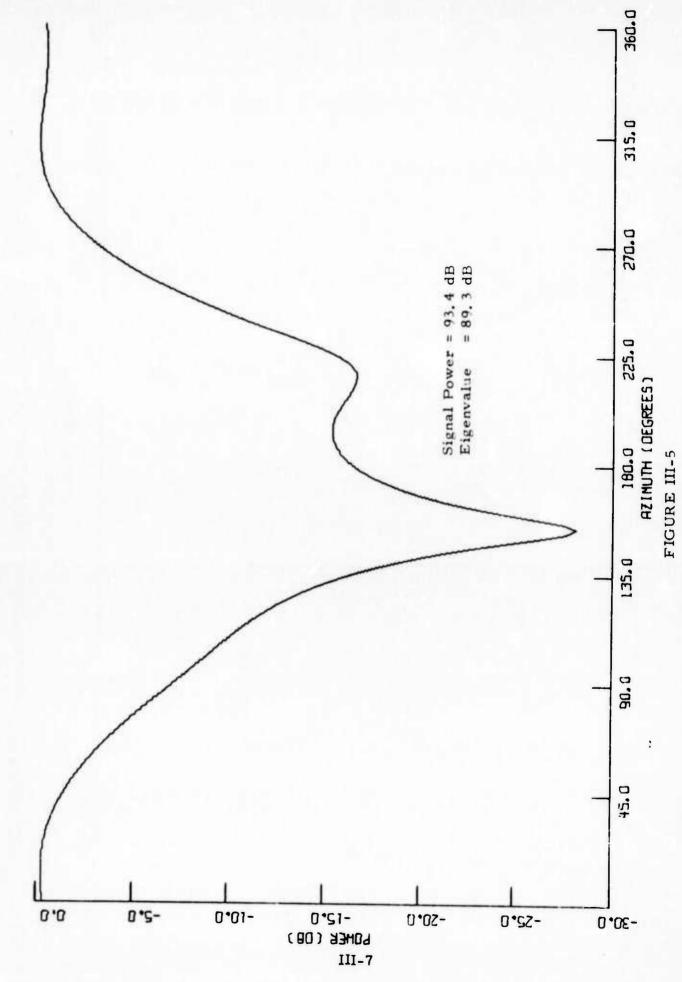
FIRST EIGENSPECTRUM FROM CRETE EVENT ALONE



SECOND EIGENSPECTRUM FROM TONGA ISLAND EVENT ALONE



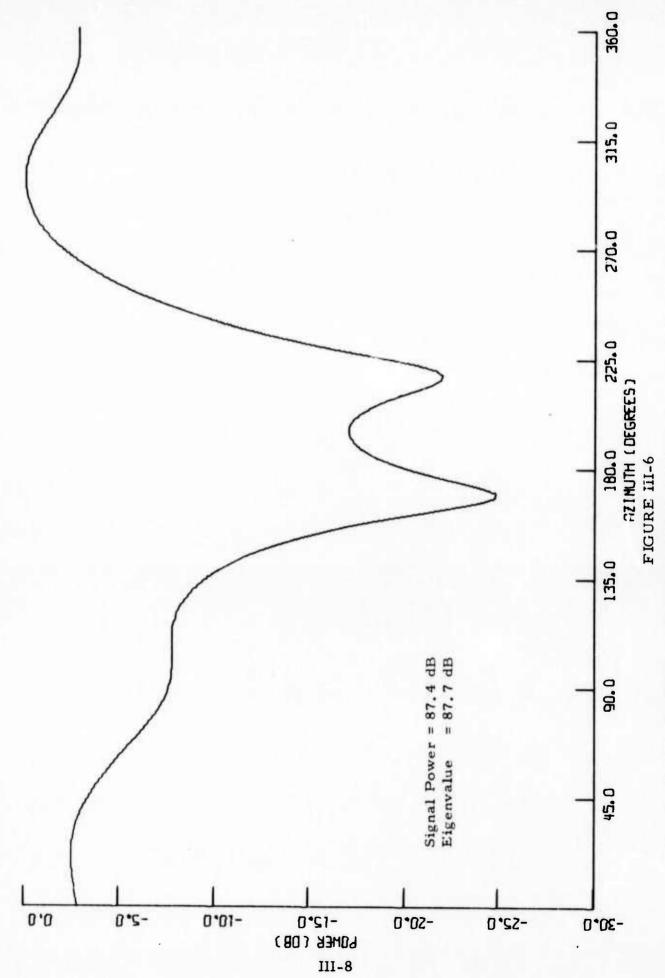
SECOND EIGENSPECTRUM FROM MIXED EVENTS AT 12 dB POWER SEPARATION



I

-

SECOND EIGENSPECTRUM FROM MIXED EVENTS AT 18 dB POWER SEPARATION



SECOND EIGENSPECTRUM FROM MIXED EVENTS AT 24 dB POWER SEPARATION

as that of Figure III-3 found in the absence of a second event. Thus at no point over the range of power studied did the eigenspectrum technique resolve the interfering signals. For small power separations the presence of the first signal leads to a large error in the estimate of the smaller event. For greater separations, the spurious peak in the second eigenspectrum obscures the true peak entirely.

In an effort to determine the cause of the incorrect estimation of the second eigenspectrum at small power separations, the data matrix was made up in a slightly different way. The matrix up to this point has been calculated by averaging the total signal vector V + W over N frequencies.

$$\Omega = \frac{1}{N} \sum_{i=1}^{N} (V_i + W_i) (V_i + W_i)^*$$
 (III-1)

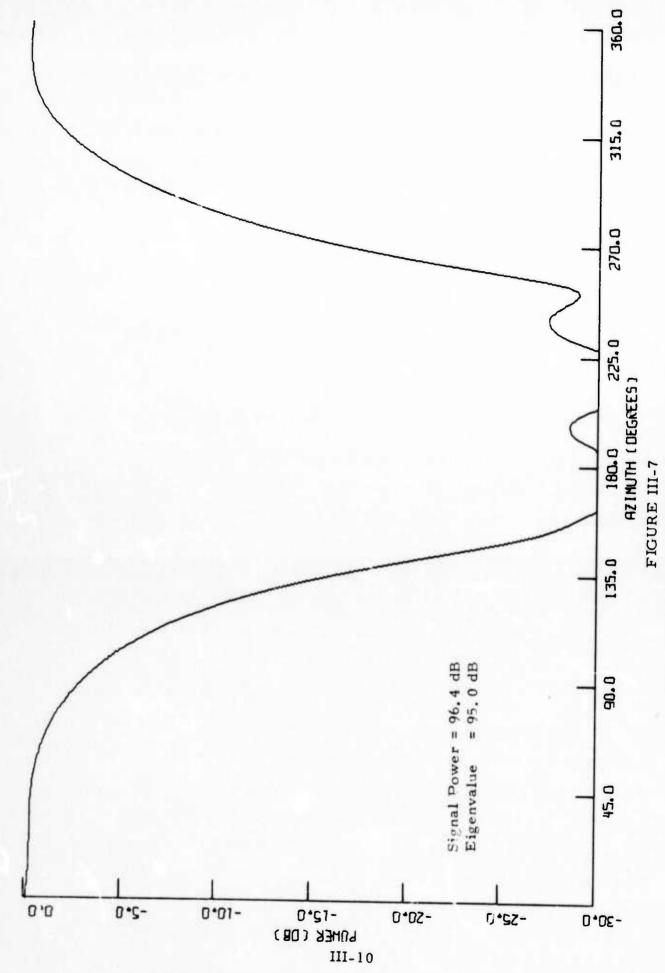
This is the only correct matrix when it is desired to simulate two signals as they are mixed naturally. Since the signals used in this study were recorded separately it was possible to construct the matrix:

$$\Omega' = \frac{1}{N} \sum_{i} v_{i}^{*} + w_{i}^{*} w_{i}^{*}$$
 (III-2)

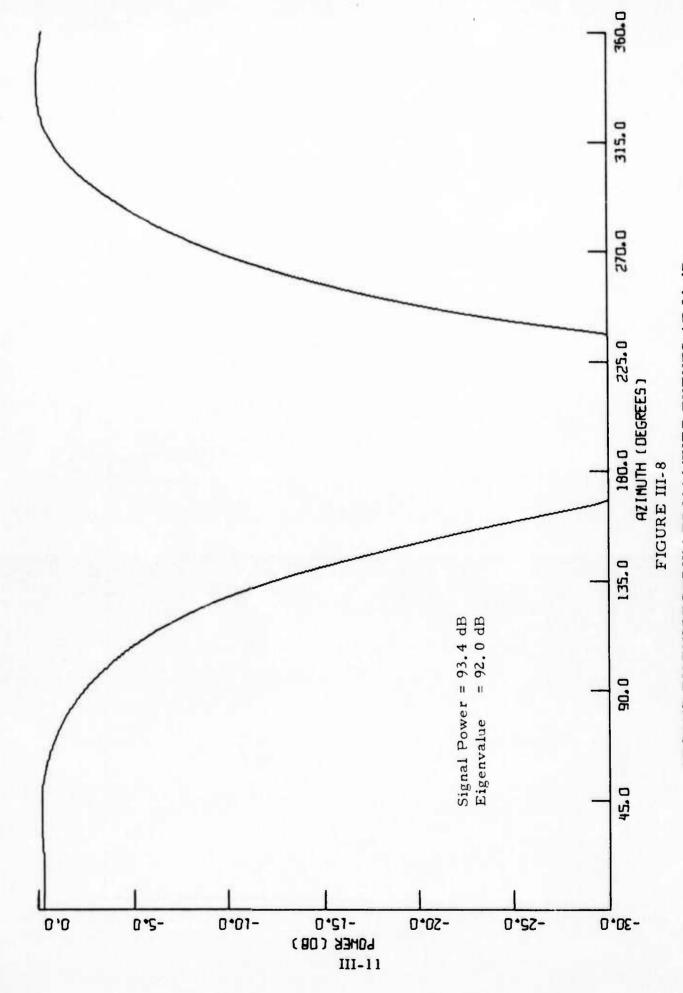
where the cross correlation matrices VW* and WV* have been left out and the data matrix corresponds to perfectly uncorrelated signals.

Typical normalized values of the cross correlation terms are $VW^*(V|W)\cong .1$. It is found that increasing the frequency averaging range does not reduce the correlation beyond a certain point, because seismic signals are truly correlated to some extent, however small.

Second eigenspectra for the signals considered above but using the data matrix of equation III-2 are shown in Figures III-7 through III-9. Although the peak of the second eigenspectrum moves further and

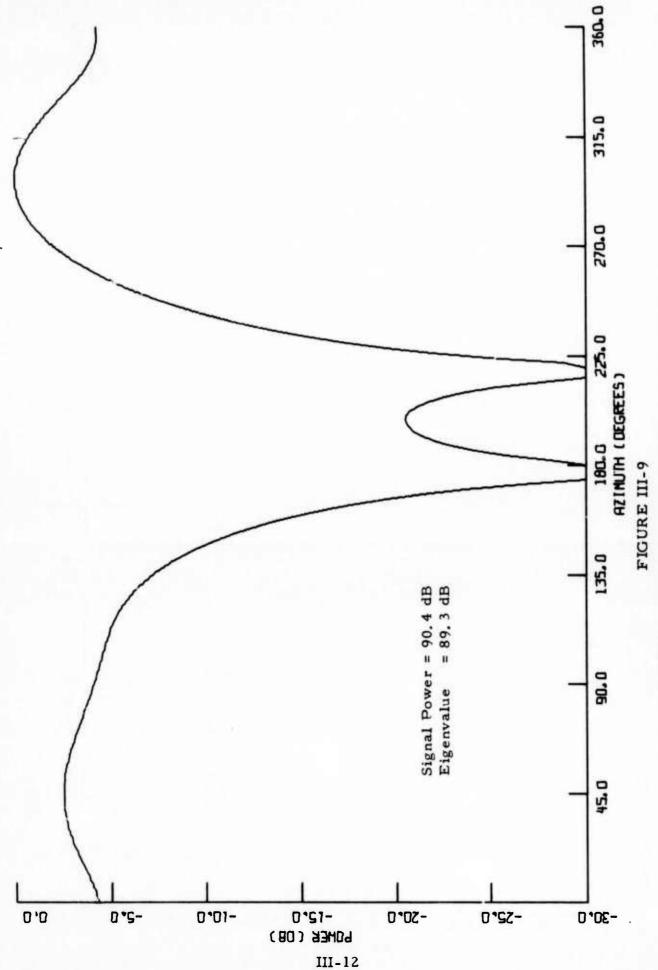


SECOND EIGENSPECTRUM FROM MIXED EVENTS AT 18 dB POWER SEPARATION - NO CROSS TERMS



I

SECOND EIGENSPECTRUM FROM MIXED EVENTS AT 21 dB POWER SEPARATION - NO CROSS TERMS



SECOND EIGENSPECTRUM FROM MIXED EVENTS AT 24 dB POWER SEPARATION - NO CROSS TERMS

further from its correct value as the power separation between the signals increases the second signal is resolved down to 18 dB separation. At 21 dB separation the azimuthal error is out of bounds, and at 24 dB separation the peak is due to the first signal alone. Thus we may conclude that correlation between the interfering signals results in the incorrect estimation of the smaller signal power, down to the point where the spurious peak in the second eigenspectrum is important.

To investigate the cause of this interfering peak the data matrix was constructed with only one frequency component and no cross terms. The second eigenvalue in the absence of a second signal lay 52 dB below the first and at the same azimuth. The second eigenvalue for interfering signals lay within 1 dB of the second signals power down to a power separation of 50 dB. This case is the analog for real data of those discussed by Booker, et al., (1972), and shows that the spurious peak arises from the necessity of averaging over frequency. If this averaging does not take place on real data, however, cross terms will be present in the matrix, and all the second signal's power will be included in the first eigenvalue.

The spurious peak in Figure III-3 is a common feature of the second eigenspectra of unmixed signals as shown by Table III-1. Here the separation, in power and azimuth, between the first and second eigenvalues of data matrices from several unmixed signals of high signal-to-noise ratio are shown. In each case the peaks in the second eigenspectra were sharp and distinct rather than broad, which would be the behavior expected if they were due to seismic noise.

C. CONCLUSIONS

In this report the effects on the eigenspectral technique of averaging the data matrix over frequency have been investigated. It is found

TABLE III-1

POWER AND AZIMUTH SEPARATION BETWEEN FIRST AND SECOND
EIGENVALUES FOR UNMIXED SIGNALS

Signal Region	Separation Between First And Second Eigenvalu	
	Power (dB)	Azimuth (degrees)
Crete	14.2	50
Kurile Islands	21.0	82
Yunan Province	19.7	58
Franz Joseph Land	24.3	88
Eastern Kazakh (Presumed Explosion)	4.9	172
Hindu Kush	19.0	152
Iran	18.2	42

that this averaging process does not result in completely uncorrelated data matrices corresponding to the first and second signals, but that non-negligible cross terms between the signals are present. These terms reflect the fact that signals of finite length are in general correlated. Their effect at small differences in power between first and second signals is to distort the second eigenvalue significantly from its correct value, although the direction of the second eigenvector is not affected.

At larger power separations between the signal powers another effect is found. There is always a spurious component in the second eigenspectrum due to the effect of averaging the signal vector of the larger signal over frequency. This component interfers with that due to the second signal when the second signal power is at or below the power of the spurious component, resulting in the incorrect estimation of the azimuth as well as the power of the second signal in this case.

Elimination of the effects of frequency averaging by constructing the data matrix in appropriate ways eliminates these problems, and allows resolution of small signals at power separations down to 50 dB. However, it is not possible to find the data matrices in these ways for interfering signals as recorded, so the problems encountered here appear insurmountable in the general case. This means that the eigenspectral technique, as presently implemented, is unable to resolve interfering signals at any useful range of power separations. Accordingly, it is recommended that no further research be carried out on this project.

SECTION IV CONCLUSIONS

A combined comb and short pass filter has been developed to aid in the resolution of interfering events at time delays comparable to the reciprocal of the signal bandwidth. Prior knowledge of the delay time and amplitude is required for the comb filter, and this may be obtained by means of a matched waveform filter.

The matched waveform filter developed here is capable of adequately estimating time delays down to 0.1 seconds, for the range of amplitudes ordinarily encountered. Using this information, direct and depth phases from presumed underground nuclear explosions may be resolved down to delay times of 0.4 seconds. At shorter time delays waveforms may be resolved but estimates of time delays are accompanied by larger estimates of error than for longer times.

The effect of frequency averaging on the estimates of power and azimuth of a small interfering event by the eigenspectrum technique has been investigated. It was found that frequency averaging, which is necessary for a stable estimate of the data matrix, is in general not adequate to completely remove the effects of cross correlation. This results in an incorrect estimate of the power of the smaller signal by this method at small power differences. At larger differences a spurious eigenvector introduced as a result of frequency averaging interfers with the estimate of the smaller eigenvalue. In this region neither the power nor the azimuth of the second signal may be resolved. The combination of these effects makes the eigenspectral technique unworkable in the general case.

SECTION V

REFERENCES

- Booker, A. H., and C. Ong, 1972, Resolution and Stability of Wavenumber Spectral Estimates, Soecial Report No. 2, Extended Array Evaluation Program, Texas Instruments Incorporated, Dallas, Texas.
- Lambert, D. G., D. H. von Seggern, S. S. Alexander, and G. A. Galat, 1969, The Long Shot Experiment, Volume II, Seismic Data Laboratory Report No. 234, AFTAC Contract Number F33657-69-C-0913, Teledyne Industries, Inc., Alexandria, Virginia
- Lane, S. S., 1973, Seimi-Annual Technical Report No. 1 Part B, ALEX(02)-TR-73-01-PART B, AFOSR Contract Number F44620-73-C-0055,
 Texas Instruments Incorporated, Dallas, Texas.
- Lane, S. S., and D. Sun, 1974, Semi-Annual Technical Report No. 2 PART B, ALEX(02)-TR-74-02-PART B, AFOSR Contract Number F44620-73-C-0055, Texas Instruments Incorporated, Dallas, Texas.